Universitatea Babeş-Bolyai Cluj-Napoca
Facultatea de Matematică şi Informatică
Ciclul de studii: Doctorat
Domeniul: Matematica
Programul de studii: Școala Doctorală de Matematică
Limba de predare: Engleză

SYLLABUS

I. General data

Code	Subject
???	**Modular Representation Theory of Finite Groups**

Semester	Hours: C+S+L	Category	Status
1	2+2+0 (Alternatively, it may be offered as a reading course)	specialty	optional

II. Full status faculty members

Name and surname	Scientific title	Didactic title	Chair	Type of activity		
				C	S	L
MARCUS Andrei	Ph.D.	Prof.	Algebra	*	*	

Associated faculty members

Name and surname	Scientific title	Institution	Type of position	Type of activity		
				C	S	L

III. Course objectives

This course is an introduction to the modular representation theory of finite groups, with a focus on the use of category theory and homological algebra. It starts by reviewing facts from ordinary representation theory, and the aim is to bring students to the point where they are able to read current research papers in the field. Applications to the study of symmetries in Physics and Chemistry are also mentioned. Homeworks include exercises of both theoretical and practical nature, and also the use of computer algebra systems, such as GAP.

IV. Course contents

Algebras and modules. Morita equivalence. Group representations and characters. Symmetric groups. Algebras over a complete local domain. G-algebras and pointed groups. Induction and relative proiectivity. Defect pointed groups and the Green Correspondence. p-permutation and endo-permutation modules. Clifford theory. Blocks of group algebras. Cartan-Decomposition triangle. Source algebra of a block. Brauer pairs. Brauer's main theorems. Blocks with normal defect groups. Derived categories and Rickard equivalences. Stable categories and stable equivalences. Blocks with cyclic defect groups. Nilpotent blocks. Fusion systems. Algorithms and GAP packages for character computations.

V. Bibliography

1. J.L. Alperin, *Local representation theory*, Cambridge Univ. Press, 1993.
2. D.J. Benson, *Representations and cohomology* I, II. Cambridge Univ. Press 1991.
3. M. Geck, D. Testerman and J. Thévenaz (eds.), *Group representation theory*. CRC Press Boca Raton, FL; EPFL Press, Lausanne, 2007.
4. K. Lux and H. Pahlings, *Representations of Groups. A Computational Approach*. Cambridge University Press, 2010.
5. A. Marcus, *Modular Representation Theory of Finite Groups. Recent Results and Open Problems*, EFES Cluj-Napoca, 2002.
6. B.E. Sagan, *The symmetric group. Representations, combinatorial algorithms, and symmetric functions*. 2nd ed. Springer-Verlag, New York, 2001.
7. J.P. Serre, *Représentations linéaires des groupes finis*, Hermann, Paris, 1971.
8. J. Thévenaz, *G-Algebras and Modular Representation Theory*, Oxford University Press, 1995.

VI. Thematic of didactic activities per weeks

Schedule of the courses and seminars

WEEK 1.

1. Algebras, modules and representations
   - 1.1. Algebras. Examples
   - 1.2. Representations and modules
   - 1.3. Abelian Categories and Morita theory
   - 1.4. The Jacobson radical. Semisimple modules
   - 1.5. Simple algebras. Brauer group. Schur Index.

WEEK 2.

2. Group representations and characters
   - 2.1. Complex representations. Maschke's theorem
   - 2.2. Characters
   - 2.3. Character values and algebraic integers
   - 2.4. Brauer's characterization of characters
   - 2.5. Characters of the symmetric group

WEEK 3.

3. Algebras over a complete local domain
   - 3.1. Complete local rings. Discrete valuation rings and p-modular systems
   - 3.2. Integral, ordinary and modular representations

WEEK 4.

   - 3.3. Lifting idempotents
   - 3.4. The Cartan matrix and the Decomposition matrix
   - 3.5. Central idempotents and blocks
   - 3.6. Projective and injective lattices
   - 3.7. Self-injective and symmetric algebras

WEEK 5.

4. Operations with kG-modules
   - 4.1. Tensor product of kG-modules
   - 4.2. Induction, restriction, coinduction
   - 4.3. Frobenius reciprocity. Mackey formulas
   - 4.4. Characters and numerical invariants of blocks
   - 4.5. Character counting conjectures

WEEK 6.
5. Relative projectivity
   5.1. Relatively projective modules. Vertices and sources
   5.2. The Green correspondence
   5.3. *p*-permutation and endo-permutation modules.
WEEK 7.
6. Clifford theory
   6.1. Group graded algebras
   6.2. The Clifford correspondence
WEEK 8.
7. G-algebras and pointed groups
   7.1. The Brauer construction
   7.2. Relative projectivity and defect pointed groups
   7.3. Source algebra of a block
WEEK 9.
8. The Brauer correspondence
   8.1. Brauer's first main theorem
   8.2. Brauer pairs.
   8.3. Brauer's 2^{nd} and 3^{rd} main theorems.
   8.4. Blocks with normal defect groups.
WEEK 10.
9. Equivalences between blocks
   9.1. Derived categories and Rickard equivalences.
   9.2. Stable categories and stable equivalences
   9.3. Broué's abelian defect group conjecture
WEEK 11.
10. Blocks with cyclic defect groups
WEEK 12.
11.Nilpotent blocks
   11.1. Local structure of a block
   11.2. Alperin's fusion theorem
   11.3. Control of fusion
WEEK 13.
12. Fusion systems (a.k.a. Frobenius categories)
WEEK 14.
13. Algorithms and GAP packages for character computations.

VII. Didactic methods used

Lectures, presentations, conversations, projects, exercises, individual study, homework assignments.

VIII. Assessment

The course ends with a written exam. The exam subjects have theoretical questions and exercises. Additionally, students will have to write a survey paper (accompanied by a beamer presentation) on a more advanced topic. There is an evaluation of the overall seminar activity (including homeworks), as well.

The final grade is the mean of the grades mentioned above, according to the following rule:

The final grade = Homeworks 30%. Paper 40%. Written Exam 30%.

IX. Additional bibliography

Prerequisites

1. J.L. Alperin and R.B. Bell, *Groups and representations*. Springer-Verlag, New York, 1995.
2. I. Assem, *Algèbres et modules*. Cours et exercices. Masson, Paris, 1997.
3. A. Marcus, *Algebra*. (in Hungarian) Cluj University Press, 2008. available at http://math.ubbcluj.ro/~marcus.
4. J.G. Rainbolt and J.A. Gallian, *Abstract Algebra with GAP*, http://math.slu.edu/~rainbolt/manual2.html
5. J.J. Rotman, *Advanced modern algebra*. 2nd ed. American Mathematical Society. Providence, RI, 2010.

Advanced

1. J.F. Carlson, *Modules and group algebras*. Birkhauser Verlag, Basel, 1996.
2. C.W. Curtis and I. Reiner, *Methods of representation theory* I, II. Interscience London-New York, 1981, 1987.
3. W. Feit, *The representation theory of finite groups*, North-Holland, Amsterdam 1982.
4. B. Huppert, *Character theory of finite groups*. Walter de Gruyter & Co., Berlin, 1998.
5. I.M. Isaacs, *Character theory of finite groups*. Dover Publications, Inc., New York, 1994.
6. G. James and M. Liebeck, *Representations and characters of groups*. 2nd ed. Cambridge University Press, New York, 2001.
7. S. König and A. Zimmermann, *Derived equivalences for group rings*, Springer-Verlag, Berlin, 1998.
8. A. Marcus, *Representation theory of group graded algebras*. Nova Science Publishers, Commack, NY, 1999.
9. G.O. Michler, *Theory of finite simple groups*. Cambridge University Press, 2006.
10. G. Navarro, *Characters and blocks of finite groups*. Cambridge University Press, Cambridge, 1998.
11. L. Puig, *On the local structure of Morita and Rickard equivalences between Brauer blocks*. Birkhauser Verlag, Basel, 1999.
12. L. Puig, *Blocks of finite groups. The hyperfocal subalgebra of a block*. Springer-Verlag, Berlin, 2002.
13. L. Puig, *Frobenius categories versus Brauer blocks. The Grothendieck group of the Frobenius category of a Brauer block*. Birkhauser Verlag, Basel, 2009.
14. The GAP Group: *GAP -- Groups, Algorithms, and Programming*. Version 4.4.3. http://www.gap-system.org

Date,
2010, September 6

Course responsible,
Prof. MARCUS Andrei, Ph.D.

DEAN,
Prof. Leon Tâmbulea, PhD.

DOCTORAL SCHOOL COORDINATOR
Prof. Adrian Petrușel, Ph.D.